Coskewness and Conditional Asset Pricing

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COMMENTS WELCOMED

I derive the beta-pricing representation of Harvey and Siddique (2000) conditional 3M-CAPM and estimate it using returns on the Fama and French (1995) 30 US industry portfolios from 1952 to 2002 and Lettau and Ludvigson (2001) consumption-wealth ratio as a conditioning variable. The parameter estimates imply an inverse S-shaped utility function and the gamma premium turns out to be insignificant when risk aversion and non-increasing absolute risk aversion are imposed. However, if we accept some risk seeking over gains, my 3-moment beta pricing representation is surprisingly successful at explaining the cross-section of average industry returns.

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1. Introduction

There is evidence that assets that display positive covariance and negative coskewness with the market portfolio tend to display higher negative average returns. Kraus and Litzenberger (1976), Friend and Westerfield (1980) and Harvey and Siddique (2000), among others, explain this empirical regularity on the basis of a three-moment extension of the Capital Asset Pricing Model (3M-CAPM). In this model, average returns depend on both a covariance and a coskewness premium.

Harvey and Siddique (2000), in particular, test the 3M-CAPM on Centre for Research on Security Prices (CRSP) NYSE, AMEX and NASDAQ equity data over the period 1963-1993. They find that the 3M-CAPM significantly improves on the 2-moment CAPM and report that coskewness is important and commands on average a risk premium of 3.6 percent per annum. Both Dittmar (2002) and Post, Levy and van Vliet (2003), however, find that the significance of the coskewness premium in the cross-section of industry and size-sorted portfolio average returns is greatly reduced when the shape of the representative investor's utility function is restricted to display non satiation, risk aversion and non increasing absolute risk aversion (henceforth, NS, RA and NIARA, respectively) over all values of sample wealth.

In this paper, I study the unconditional implications of both conditional beta and beta *and* gamma pricing models with conditionally time varying utility function parameters and possible preference for portfolio skewness. In the next section, I present the 3-Moment conditional CAPM (3M-CCAPM) formulated by Harvey and Siddique (2000) and I derive its beta pricing representation. The latter is, to my knowledge, novel. In Section 3, I specify an empirical version of this beta pricing representation. In Section 4, I introduce the alternative beta-gamma representation of the 3M-CAPM proposed by Kraus and Litzenberger (1976) and extended in a conditional setting by Dittmar (2002). In Section 5, I present my dataset. Then, in Section 6 and 7, I estimate both specifications with and without restrictions by, respectively, a 2-pass procedure that uses system 3-stage least squares (system 3SLS) and generalised method of moments (GMM). I also check that the NS, RA and

NIARA conditions are satisfied. In Section 8, I discuss the asset pricing implications of risk-seeking. In Section 9, I summarize my findings and present my conclusions.

2. The Conditional 3M-CAPM

Denoting by f_{t+1} the factors that drive marginal utility, I start from a linear formulation of the stochastic discount factor m_{t+1} of the representative investor:

$$m_{t+1} = a_t + b_t' f_{t+1} \tag{1}$$

Harvey and Siddique (2000) approximate the stochastic discount factor m_{t+1} as a quadratic function of the market return $R_{m,t+1}$. Since third degree polynomial utility functions are unsuitable to model the preferences of a risk adverse investor, as shown by Levy (1969) and Tsiang (1972), this specification is based on a third order Taylor expansion of a non polynomial utility function. Assuming the existence of a conditionally risk-free asset and letting $f_{t+1} = [r_{m,t+1} \quad r_{m,t+1}^2]'$, it implies the following conditional 3 moment factor model:

$$m_{t+1} = a_t + b_{1t} r_{m,t+1} + b_{2t}\, r_{m,t+1}^{2} \tag{2}$$

Arrow (1971) argued that desirable properties for an investor's utility function $u(W)$ defined over wealth W are positive and decreasing marginal utility of wealth and NIARA. Positive marginal utility of wealth, or $u' > 0$, implies investors' NS whereas decreasing marginal utility of wealth, $u'' < 0$, implies RA. NIARA, as shown in Arditti (1967), is a sufficient condition for $u''' > 0$ and hence it implies preference for positive skewness. These properties imply restrictions on the representative investor's stochastic discount factor.

Let $r_{i,t}$ denote the excess-return on the asset i. Under the law of one price, I can impose the Euler equation equilibrium conditions $0 = E_t(m_{t+1}, r_{i,t+1})$, every asset i, for the maximization of a 2-period representative investor's utility. When pricing excess returns, the coefficient a_t can be normalized arbitrarily. Hence, as recommended by Cochrane (2001), I set $a_t = 1 - b_t' E_t(f_{t+1})$. Therefore, I can write $m_{t+1} = 1 + b_t' \left[f_{t+1} - E_t(f_{t+1}) \right]$. I can then write the conditional 3M-CAPM asset pricing equation proposed by Harvey and Siddique (2000) and its beta pricing representation as follows:

$$E_t(r_{i,t+1}) = - b_t' Cov_t(f_{t+1}, r_{i,t+1})$$

$$= - b_{1,t}\, Cov_t(r_{i,t+1}, r_{m,t+1}) - b_{2,t}\, Cov_t(r_{i,t+1}, r_{m,t+1}^2) \tag{3}$$

$$E_t(r_{i,t+1}) = \beta_{it}' \lambda_t \tag{4}$$

Where:

$$\beta_{i1,t} = \frac{Cov_t(r_{i,t+1}, r_{m,t+1})Var_t(r_{m,t+1}^2) - Skew_t(r_{m,t+1})Cov_t(r_{i,t+1}, r_{m,t+1}^2)}{Var_t(r_{m,t+1})Var_t(r_{m,t+1}^2) - [Skew_t(r_{m,t+1})]^2}$$

$$\beta_{i2,t} = \frac{Cov_t(r_{i,t+1}, r_{m,t+1}^2)Var_t(r_{m,t+1}) - Skew_t(r_{m,t+1})Cov_t(r_{i,t+1}, r_{m,t+1})}{Var_t(r_{m,t+1})Var_t(r_{m,t+1}^2) - [Skew_t(r_{m,t+1})]^2}$$

$$\lambda_t = - E_t(f_{t+1}' f_{t+1})\, b_t$$

$$= - \begin{bmatrix} Var_t(r_{m,t+1}) & Cov_t(r_{m,t+1}, r_{m,t+1}^2) \\ Cov_t(r_{m,t+1}, r_{m,t+1}^2) & Var_t(r_{m,t+1}^2) \end{bmatrix} \begin{bmatrix} b_{1,t} \\ b_{2,t} \end{bmatrix}$$

Here, the symbol $Skew_t(r_{m,t+1}) = E_t\left\{ \left[r_{m,t+1} - E_t(r_{m,t+1}) \right]^3 \right\}$ represents the conditional skewness of the market portfolio and $r_{i,t}$ denotes the excess-return on the asset i. The factor loadings, the parameters $\beta_{i1,t}$ and $\beta_{i2,t}$, are functions of the market variance, its skewness and of the covariance and coskewness of the asset i with the market. They are both risk measures and coefficients of the regression of the asset excess-return on the market excess return and its square. From this point of view, they are analogous to the CAPM beta coefficient. Equation (4) is a testable restriction that the model in

(2) imposes on the cross section of expected (average) asset returns. The upshot of this formulation relative to the specification proposed by Harvey and Siddique (2000) is that I can test the unconditional implications of the conditional 3M-CAPM for the cross section of asset returns using a simple two pass estimation procedure, by regressing asset excess returns on the factors and the average excess returns on the estimated factor loadings.

3. A Conditional 3M-CAPM Empirical Specification

While Harvey and Siddique's (2000) 3M-CAPM implies the conditional stochastic discount factor model in (2), they estimate it without allowing for time variation in the parameters of the investor's marginal utility. Therefore, Harvey and Siddique (2000) effectively estimate an unconditional rather than a conditional 3M-CAPM, even though they do allow for time variation in betas. To formulate an empirical specification of the conditional 3M-CAPM in (2) that allows for time variation in the parameters of the investor's marginal utility, I let the a_t and b_t parameters of the stochastic discount factor vary as a linear function of the conditioning information provided by the variable z_t, $a = a^0 + a^1 z_t$, $b_{1,t} = b_1^0 + b_1^1 z_t$, $b_{2,t} = b_2^0 + b_2^1 z_t$. Treating the vector $f_{t+1} = [z_t \;\; r_{m,t+1} \;\; z_t r_{m,t+1} \;\; r^2_{m,t+1} \;\; z_t r^2_{m,t+1}]'$ as the factors, this implies the following beta-pricing representation:

$$r_i = \alpha + \beta_{i3}\lambda_3 + \beta_{i4}\lambda_4 + \beta_{i5}\lambda_5 + \beta_{i6}\lambda_6 + \beta_{i7}\lambda_7 + \varepsilon_i \qquad (5)$$

Here, the elements of the λ vector are the cross-sectional parameter estimates of average asset excess returns on the corresponding elements of the β vector. The latter are the parameters estimates of the following time series regressions:

$$r_{it} = \alpha_i + \beta_{i3} z_{t-1} + \beta_{i4} r_{mt} + \beta_{i5} z_{t-1} r_{mt} + \beta_{i6} r_{mt}^2 + \beta_{i7} z_{t-1} r_{mt}^2 + \varepsilon_{it} \qquad (6)$$

I allow for an intercept in (5) and (6). A model that is fully successful at explaining the cross section of asset excess returns should have $\alpha = 0$.

4. Beta-Gamma Conditional 3M-CAPM

The 3M-CAPM specification proposed by Kraus and Litzenberger (1976) is based on the Euler equation for a (standardised[1]) cubic approximation of a non-polynomial utility function $u(x) = x + \theta_1 x^2 + \theta_2 x^3$, i.e. $E[(1 + 2\,\theta_1\, r_m + 3\,\theta_2 r_m^2) r_i] = 0$. I drop time subscripts for notational simplicity. This is equivalent to (2) with $a_1 = 1$, $b_1 = 2\,\theta_1$ and $b_2 = 3\,\theta_2$. In the conditional version of the 3-Moment Capital Asset Pricing Model (3M-CCAPM), I account for time-variation in the parameters of the utility function letting them depend in a linear fashion on a conditioning variable z that represent the available information set:

$$\theta_1 = \theta_1^0 + \theta_1^1 z$$
$$\theta_2 = \theta_2^0 + \theta_2^1 z$$
(7)

I then have $u(x) = x + (\theta_1^0 + \theta_1^1 z)x^2 + (\theta_2^0 + \theta_2^1 z)x^3$, i.e. $u(x) = x + \theta_3\, x^2 + \theta_4\, z\, x^2 + \theta_5\, x^3 + \theta_6\, z\, x^3$. The corresponding Euler equations that give the unconditional asset pricing implications of the conditional discount factor model are, for every asset i:

$$E[u'(r_m)\, r_i] = E[(1 + 2\,\theta_3\, r_m + 2\,\theta_4\, z r_m + 3\,\theta_5 r_m^2 + 3\,\theta_6\, z r_m^2) r_i\,] = 0 \qquad (8)$$

Using a Taylor expansion of the first derivative of the utility function around the point corresponding to the expected market excess return, Post, Levy and van Vliet (2003) derive the risk premia δ_1 and δ_2 from the utility function parameters estimates as follows:

$$\delta_1 = \frac{-E[u''(r_m \mid \theta)]E[r_m - E(r_m)]^2}{E[u'(r_m \mid \theta)]} \qquad (9)$$

$$\delta_2 = \frac{-0.5E[u'''(r_m \mid \theta)]E[r_m - E(r_m)]^3}{E[u'(r_m \mid \theta)]} \tag{10}$$

Plugging (7) into (9) and (10), I then recover the conditional risk premia δ_1 and δ_2 from the conditional utility function parameters as follows:

$$\delta_1 = \frac{-[2\theta_3 + 2\theta_4 E(z) + 6\theta_5 E(r_m) + 6\theta_6 E(zr_m)]E[(r_m - E(r_m)]^2}{1 + 2\theta_3 E(r_m) + 2\theta_4 E(zr_m) + 3\theta_5 E(r_m^2) + 3\theta_6 E(zr_m^2)}$$

(11)

$$\delta_2 = \frac{-[3\theta_5 + 3\theta_6 E(z)]E[(r_m - E(r_m)]^3}{1 + 2\theta_3 E(r_m) + 2\theta_4 E(zr_m) + 3\theta_5 E(r_m^2) + 3\theta_6 E(zr_m^2)}$$

(12)

The above equations give the unconditional asset pricing implications in terms of the two risk factors, the market excess return and its square, of the conditional discount factor model based on (8). Under non satiation, marginal utility must always be positive. Therefore the denominator of (9) is always positive. Since variance is always positive, risk aversion and hence $u''(r_m \mid \theta) \leq 0$ implies $\delta_1 \geq 0$. Similarly, since the skewness of the market portfolio is empirically found to be negative, preference for skewness and hence NIARA and $u'''(r_m \mid \theta) \geq 0$ implies a positive δ_2, i.e. $\delta_2 \geq 0$.

5. Data

I construct quarterly observations from the 3rd quarter of 1952 to the 3rd quarter of 2002 on the returns on the 30 Fama and French (1995) US industry portfolios and on the market portfolio using the Centre for Research on Security Prices (CRSP) database. I also use quarterly returns on the 3-month US Government Treasury Bill as a proxy for the risk free rate. I denote the risk-free rate proxy as r_{ft}. The expressions r_{it} denote excess-returns on the industry portfolios. Excess returns are computed as the difference between returns and the risk free rate. Finally, I denote by

cay_t the consumption-wealth ratio per capita estimates produced by Lettau and Ludvigson (2001).

6. Two-Pass Regression Estimates

Using cay_t as the conditioning variable, I set $z_t = cay_t$ in (5). The model in (5) and (6) can be estimated by a 2-pass procedure that involves time series and cross-sectional regressions. In the first pass, I estimate in a maximum likelihood setting, by three stage least squares (3SLS), the system of time-series regressions equations in (6) for the industries in my sample. I do not impose any constraint on the contemporaneous covariance of the residuals nor on their variance. I do correct, however, the variance and covariance matrix of the estimates for possible error autocorrelation and heteroskedasticity. In the second pass of the estimation procedure, I then use my estimated beta coefficients as the regressors of average industry excess returns in a cross-sectional regression based on (5). The coefficient of determination R^2 is slightly above 31 percent (17 percent adjusted).

Does the conditional model improve on the unconditional 3M-CAPM? The R^2 of the latter is slightly lower, almost 28 percent, but it is larger once we adjust for the degrees of freedom (22 percent). All the coefficient estimates are statistically significant. The CCAPM performs considerably worse than the conditional and unconditional 3M-CAPM. Its R^2 is just 7.5 percent (the adjusted one is marginally negative) and none of the coefficients estimates, with the exception of the intercept, are statistically different from zero at conventional significance levels. These results are summarised in Panel A of Table 1. They provide evidence that systematic asset co-skewness does help explain the cross-section of average returns. Even explicitly allowing for conditional time-variation in the shape of the utility function does not drive out its cross-sectional explanatory power.

There is a problem, however. Both the 3M-CAPM and the 3M-CCAPM estimates imply a shape of the utility function that is incompatible with the risk aversion requirement. This can be seen by solving (4) for the parameters of m_{t+1}, given estimates of λ_t and $E_t(f'_{t+1}f_{t+1})$:

$$b_t = - E_t(f'_{t+1}f_{t+1})^{-1} \lambda_t \tag{13}$$

I report in Figure 2 the stochastic discount factor m_{t+1} implied by the 2M-CCAPM, the 3M-CAPM and the 3M-CCAPM parameter estimates. These are consistent in all three cases with investors' non-satiation and preference for skewness. However, only the 2M-CCAPM stochastic discount factor displays risk aversion for every value taken by the market excess return over the sample. Both the 3M-CAPM and the 3M-CCAPM parameter estimates imply risk aversion only over excess returns below 1.5 percent. Above this threshold, the shape of the estimated stochastic discount factor implies risk seeking. In other words, these estimates imply an inverse S-shaped utility function.

7. GMM Estimates with RA and NIARA

To impose the RA and NIARA conditions, I estimate the 3M-CAPM specification proposed by Kraus and Litzenberger (1976) by GMM. In general, RA implies:

$$u''(r_m) = 2\,\theta_3 + 2\,\theta_4\, z + 6\,(\theta_5 + \theta_6\, z)\, r_m \leq 0 \tag{14}$$

A cubic utility function cannot be concave over its entire domain. Rather, it suffices to require the condition in (14) to hold over the sample values of r_m and z. This condition is in general difficult to impose. However, when NIARA holds:

$$u'''(r_m) = 6(\theta_5 + \theta_6\, z) \geq 0 \tag{15}$$

In this case a sufficient condition for RA is the following:

$$2\,\theta_3 + 2\,\theta_4\, z + 6\,[\theta_5 + \theta_6\, Max(z)]\, Max(r_m) \leq 0 \tag{16}$$

Here, the operator *Max*() denotes the sample maximum of the argument. Finally, the following condition adapted from Post, Levy and van Vliet (2003) constraints the market premium to be the sum of the beta and gamma premium:

$$E\left[u'(r_m)r_i\right]' w = E\left[(1 + 2\,\theta_3\, r_m + 2\,\theta_4\, zr_m + 3\,\theta_5 r_m^2 + 3\,\theta_6\, zr_m^2)r_i\right]' w$$
$$= E(r_m) + 2\,\theta_3 E(r_m^2) + 2\,\theta_4\, E(z^2 r_m^2) + 3\,\theta_5\, E(r_m^3) + 3\,\theta_6\, E(z^3 r_m^3) = 0 \quad (17)$$

Here, w represents the vector of value-weights (which can be time-varying). In the empirical applications, I replace the unconditional expectations by the corresponding sample moments and I estimate them by GMM. Within a GMM framework, I run my estimation with and without instruments (the constant and up to four lags of the market excess return). Without instruments, I estimate the system of 30 orthogonality conditions in (8) with the constraint in (17) and with and without the constraints in (15) and (16) by multivariate non-linear system least squares. With the instrumental variables, the set of orthogonality conditions is expanded to include the orthogonality of the pricing errors from (8) and each of the instruments. I estimate this system by GMM-IV with a continuously updating weighting matrix for the orthogonality conditions.

Turning to the empirical results, reported in Panel B of Table 1, when I impose the RA and NIARA constraints, the gamma premium becomes close to zero. This confirms the result reported by Dittmar (2002) and by Post, Levy and van Vliet (2003). It is interesting to note, however, that it is the NIARA restriction to be binding, whereas the RA constraint is slack. In other words, the only way to allow for a non-zero gamma premium is to allow for a negative third derivative of utility, i.e. $u'''(r_m) < 0$. This, however, implies from (12) a negative gamma premium, skewness aversion instead of skewness preference and an S-shaped utility function. It would also be interesting to impose NS and NIARA without imposing RA to replicate the results of the 2-pass regression. This, however, is virtually impossible as it is very difficult to impose NS without first imposing RA and imposing the latter without first imposing NIARA.

8. Risk Seeking

Numerous contributions from the literature on non standard utility theory and behavioural asset pricing, see for a review Shefrin (2005), admit a non linear pricing kernel that implies non concavity of the utility function over certain ranges of wealth. Similarly, active stock traders appear to play negative-sum games and their behavior can sometimes be interpreted as 'gambling' (see Statman (2002)). In addition, psychologists led by Kahneman and Tversky (1979) find experimental evidence for local risk seeking behavior. More specifically, Post and Levy (2002) argue that a number of celebrated asset pricing anomalies, such as the low average yield on stocks with large capitalization, growth stocks and past winners, could be explained by risk aversion over losses and risk seeking over gains. Friedman and Savage (1948) and Markowitz (1952) argue that the willingness to purchase both insurance and lottery tickets implies that marginal utility is increasing over a range. See Hartley and Farrell (2001) and Post and Levy (2002) for a recent discussion.

Notice that, in a constrained optimization problem, a stationary point is guaranteed to represent a maximum only when the objective function is quasi-concave. Both inverse S-shaped and S-shaped utility functions are (strictly) quasi-concave. However, the sum of quasi-concave functions (unlike the sum of concave ones) is not guaranteed to be quasi-concave[2]. Hence, the parameters of the representative investor's stochastic discount factor that satisfy the first order conditions $0 = E_t(m_{t+1}, r_{i,t+1})$ are not guaranteed to represent the constrained maximum of his expected utility function. Similarly, there is no guarantee that the stochastic discount factor parameters are such that the market portfolio maximizes the representative investor's expected utility, even though the first order conditions $0 = E_t(m_{t+1}, r_{i,t+1})$ are satisfied. In other words, non-concave utility functions, unlike concave ones, do not imply that the market portfolio is efficient for the representative investor. This then rises the problem of motivating why (2) specifies m_t as a function of the market excess return and its square. In particular, the interesting question then becomes why the latter should be good proxies for aggregate marginal utility growth even though the market portfolio is not necessarily efficient for the representative investor.

We might tentatively proceed as follows. We might appeal to a theorem credited to Harrison and Krepp (1979) to motivate a stochastic discount factor representation of the asset pricing problem without requiring that the market portfolio maximizes investors' expected utility. This theorem states that, given free portfolio formation and under the law of one price, there exists an m_{t+1}^j such that, for every payoff x_{t+1}, $v_t^j = E(m_{t+1}^j x_{t+1}|I_t^j)$. Here, v_t^j is the value assigned by investor j to the a payoff x_{t+1} paid in t+1. Also, I_t^j is the information set available to the investor j at time t and the expectation is taken conditional on it. Hansen and Richard (1987) prove that, under the same assumptions, m_{t+1}^j is unique and that, in the absence of arbitrage, it is positive. In market equilibrium, when asset demand equals supply, the pricing equation $v_t^j = E(m_{t+1}^j x_{t+1}|I_t^j) = p_t$ must hold for every rational investor. Under appropriate conditions, such as the assumptions that investors' expectations are rational and that their information is homogenous, it is possible[3] to find a process m_{t+1}, called the stochastic discount factor, that satisfies this equation for all investors, i.e. $E(m_{t+1}^j x_{t+1}|I_t^j) = E_t(m_{t+1} x_{t+1}) = p_t$, every j. The process m_t is in general a convex combination of the individual investors' m_{t+1}^j processes.

We can interpret m_{t+1}^j as being proportional to the marginal utility growth of investor j. Hence, by construction, m_t is proportional to aggregate marginal utility growth. On the basis of a third order Taylor expansion of investors' utility functions defined over their own wealth, we can also let m_{t+1}^j depend on the excess return on investor j own portfolio of risky assets and its square, $m_{t+1}^j = a_t^j + b_{1t}^j r_{j,t+1} + b_{2t}^j r_{j,t+1}^2$. It is then conceivable that, by aggregation, a (conditionally) linear function of the market excess returns and its square provides a reasonably good proxy for the aggregate stochastic discount factor m_{t+1} even though the market portfolio is not the representative investor's efficient combination of risky assets. The interesting issue then becomes the specification of the exact conditions under which this is guaranteed to be the case. I leave this, however, for future research. Notice that m_{t+1} cannot be the representative investor's stochastic discount factor if, given m_{t+1}, the market portfolio does not maximize his expected utility and, hence, it is not his efficient combination of risky assets. This implies that asset prices are not set by a representative investor.

9. Main Findings and Conclusions

In this paper I update the evidence provided by Harvey and Siddique (2000) and by Dittmar (2002) on the ability of the coskewness and gamma premia to explain the cross-section of US industry returns. My sample spans 50 years from 1952 to 2002, whereas the sample period of the studies of Harvey and Siddique (2000) and Dittmar (2002) stops, respectively, in 1993 and 1995. I also employ cay_t as a conditioning variable to model time variation in the parameters of the utility function. This variable had not been used by Dittmar (2002). Relative to Harvey and Siddique (2000), the main innovation of this study is an explicitly conditional empirical specification of the stochastic discount factor and the derivation of the beta-pricing representation of their model. My beta pricing representation of Harvey and Siddique (2000) 3M-CCAPM is also different from Kraus and Litzenberger (1976) beta-gamma representation since neither their betas nor their gammas are regression coefficients. Their beta and gamma premium must be recovered estimating the utility function parameters.

If we accept some risk seeking over gains and an inverse S-shaped utility function, my beta pricing representation of Harvey and Siddique (2000) 3M-CCAPM is surprisingly successful at explaining the cross section of industry returns, with a coefficient of determination between 20 and 30 percent. These values are high for a model that does not include among the regressors portfolios returns that mimic additional and partially ad-hoc factors such as size and the book to market ratio. My results show that Markowitz type utility functions, with risk aversion for losses and risk seeking for gains, can be successful at capturing the cross-section of stock returns, once we allow for investors' twin desire for downside protection in bear markets and upside potential in bull markets. Further research might suitably expand the set of conditioning variables to better model variation in the utility function parameters. This, while improving the fit of the model, might even lead to find a specification that requires a milder violation of RA.

Table 1
3M-CAPM, CCAPM and 3M-CCAPM

Panel A
(Two-Pass Regression Estimates)

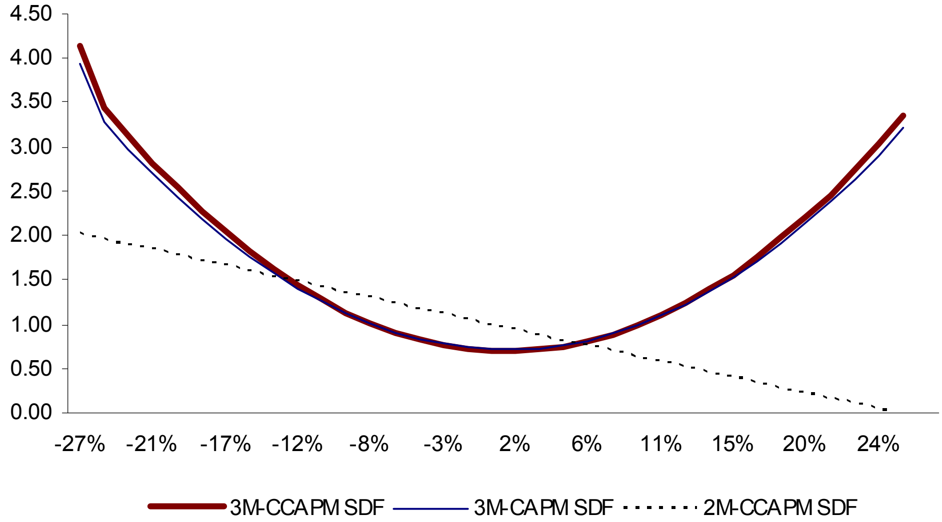
	α	λ_3	λ_4	λ_5	λ_6	λ_7	R^2 ($Adj.\ R^2$)
3M-CCAPM	1.06% [.019]	-.07% [.714]	.80% [.069]	-.01% [.091]	-.54% [.000]	.003% [.009]	31.17% (16.83%)
3M-CAPM	1.06% [.013]		.89% [.023]		-.50% [.000]		27.87% (22.52%)
CCAPM	1.22% [.010]	.30% [.233]	.72% [.105]	.00% [.581]			7.49% (-3.18%)

Panel B
(GMM Estimates)

Model	Constr.	*Estimat.*	*DF*	*TJ*	θ_3	θ_4	θ_5	θ_6	δ_1 (%)	δ_2 (%)
3M-CAPM		GMM	28	32.48 [.255]	-1.26 [.000]	0	-7.66 [.000]	0	6.95	-1.3
3M-CAPM	*RA + NIARA*	GMM-IV	88	33.91 [.999]	-1.16 [.000]	0	.113 [.000]	0	5.63	.0
3M-CCAPM		GMM	26	34.34 [.126]	-1.08 [.000]	-33.01 [.321]	-8.28 [.000]	114.8 [.322]	9.99	-4.0
3M-CCAPM	*RA + NIARA*	GMM	26	60.55 [.000]	-1.14 [.000]	-106.9 [.000]	0.00 [.000]	277.0 [.013]	5.43	.0

Notes. Panel A of this table reports coefficient estimates and measures of fit (R^2 and adjusted R^2) for the 2-pass estimation of the conditional and unconditional 3M-CAPM and of the CCAPM. Panel B reports the GMM estimation results for various set of orthogonality conditions that correspond to the 3M-CAPM (top two rows) and to the 3M-CCAPM (lower two rows). *DF* denotes degrees of freedom (number of orthogonality conditions in excess of the number of parameters to be estimated). The expression *TJ* is *T* (the sample size) times Hansen's (1982) *J* statistic and it is distributed as a Chi-Squared with degress of freedom equal to the number of over-identifying restrictions (*DF*). All the other variables are defined as in the text. The risk premia δ are annualised. Significance levels of t-statistics appear in brackets. The sample period is 1952-2002.

Figure 1
Estimated SDF



Notes. This Figure plots the estimated stochastic discount factor for the 3M-CCAPM, the 3M-CAPM and the 2M-CCAPM. The estimation used a 2-pass procedure with 3-Stage OLS estimates for the first step. The sample period is 1952-2002.

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[1] This utility function is standardized, following Post, Levy and Van Vliet (2003), such that $u(0|\theta) = 0$ and $u'(0|\theta) = 1$. Since utility functions are unique up to a linear transformation, this standardization does not affect our results. This convenient standardisation is possible, as discussed by Cochrane (2001), when working with excess returns because the relevant Euler equations do not identify the mean of the stochastic discount factor.

[2] This is not the case for concave functions. The sum of concave functions is guaranteed to be concave. This is why the concavity of utility guarantees the concavity of the expected utility function.

[3] If investors did not use the same conditioning information set and they did not process it in the same way the solution for m_t that ensures that $v_t^j = p_t$, every j, is not necessarily unique. In general, every investor could have a different m_t^j.